

Some thoughts about protractors for vinyl reproduction

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1 Mathematical description

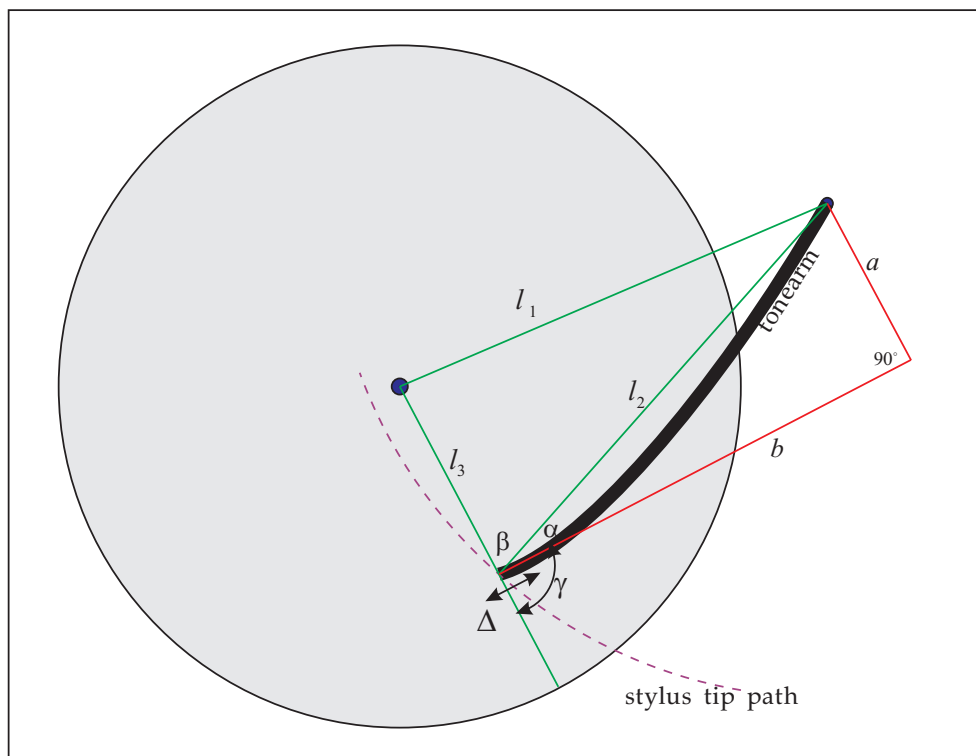


Figure 1: My mathematical turntable.

Where:

- l_1 = the distance from the center of turntable plateau to the tone arm pivot
- l_2 = the effective arm length
- l_3 = the distance from the center of the turntable plateau to the needle
- $x = \frac{l_2}{l_1}$
- a = the offset length
- b = the base of the triangle formed by l_2 and a
- Δ = the extension of b in the headshell, approximately 10 mm
- α = the offset angle of the cantilever
- β = the angle between l_2 and l_3
- γ = tracking angle

One can conclude from Fig. 1 directly

$$\begin{cases} l_2 = \sqrt{a^2 + (b + \Delta)^2} \\ l_2 = \frac{b + \Delta}{\cos \alpha} \end{cases} \Leftrightarrow \alpha = \arccos \left[\frac{b + \Delta}{\sqrt{a^2 + (b + \Delta)^2}} \right], \quad (1)$$

$$\gamma = 180^\circ - \alpha - \beta \quad (2)$$

and

$$l_1^2 = l_2^2 + l_3^2 - 2l_2l_3 \cos \beta \quad (3)$$

, where it can be seen that the extension in the headshell has little effect on the offset angle α . Let us assume that l_1 , a and b are constants¹. By doing so, one defines Δ and the playing radius l_3 to be the main variables. The main question is now which value of Δ should one select for optimum performance.

¹This is done based on tone arm geometry.

1.1 Calculation of the zero's

It can directly be seen that if a tangential point is reached, the tracking angle $\gamma = 90^\circ$. This means that $\alpha + \beta = 90^\circ$. Substituting this into Eq. (3) and (1) and solving for the radius l_3 , one can write

$$n_{1,2} = a \pm \sqrt{l_1^2 - (b + \Delta)^2} \quad (4)$$

or, written in terms of offset error and effective arm length

$$n_{1,2} = l_2(\Delta) \sin[\alpha(\Delta)] \pm \sqrt{l_1^2 [1 - x^2 \cos^2[\alpha(\Delta)]]} \quad (5)$$

Where it can be seen that

$$x < \frac{1}{\cos[\alpha(\Delta)]} \quad (6)$$

Eq. (4) can be rewritten if there are two zero's known at forehand ($n_1 < n_2$). For the calculation of a and b :

$$\begin{cases} a = \frac{n_1 + n_2}{2} \\ b = \frac{\sqrt{4l_1^2 - (n_2 - n_1)^2}}{2} - \Delta \end{cases} \quad (7)$$

And for the calculation of a and l_1 :

$$\begin{cases} a = \frac{n_1 + n_2}{2} \\ l_1 = \frac{\sqrt{4(b + \Delta)^2 + (n_2 - n_1)^2}}{2} \end{cases} \quad (8)$$

Sometimes, l_1 and l_2 are given by the manufacturer. For this case, the following equations can be handy:

$$\begin{cases} a = \frac{n_1 + n_2}{2} \\ \alpha = \arccos \left[\frac{\sqrt{4l_1^2 - (n_2 - n_1)^2}}{2l_2} \right] \\ b = \frac{\sqrt{4l_1^2 - (n_2 - n_1)^2}}{2} - \Delta \end{cases} \quad (9)$$

2 Choice of zero points, peak distortion equivalence

Here, the zero's are predetermined and found to be

$$\frac{2}{n_1} = \frac{1 + \sqrt{\frac{1}{2}}}{r_1} + \frac{1 - \sqrt{\frac{1}{2}}}{r_2}, \quad (10)$$

$$\frac{2}{n_2} = \frac{1 - \sqrt{\frac{1}{2}}}{r_1} + \frac{1 + \sqrt{\frac{1}{2}}}{r_2},$$

where r_1 is the inner radius where the track modulation ends and r_2 denotes the outer radius, where the modulation starts.

Eq. (10) will result in zero's at 66 mm and 120.89 mm for IEC records and 63.1 mm and 119.17 mm for records according to the DIN standard.

Where the inner and outer radius can be summarized as:

	inner radius [mm]	outer radius [mm]
IEC	60.325	146.05
DIN	57.5	146.05

3 Tracking error

The tracking angle is found to be

$$\gamma = 180^\circ - \arccos \left[\frac{b + \Delta}{\sqrt{a^2 + (b + \Delta)^2}} \right] - \arccos \left[\frac{a^2 - l_1^2 + l_3^2 + (b + \Delta)^2}{2l_3 \sqrt{a^2 + (b + \Delta)^2}} \right] \quad (11)$$

The error is simply the (absolute) deviation from 90° .

3.1 Fixed offset angle

An interesting property of the tone arm is the tracking error in terms of the deviation from the tangential case. If one sets the offset angle to a certain value, one can calculate l_1 and x . If this is done, the complete physical description is known. For an offset α one can calculate:

	α [deg]	l_1 [mm]	x	l_2 [mm]
IEC	24	211.67	1.0854	229.66
DIN	24	206.60	1.0845	224.06

Where

$$l_1 = \frac{\sqrt{(n_1 + n_2)^2 - 4n_1n_2 \sin^2(\alpha)}}{2 \sin(\alpha)} \quad (12)$$

and

$$x = \frac{n_1 + n_2}{\sqrt{(n_1 + n_2)^2 - 4n_1n_2 \sin^2(\alpha)}} \quad (13)$$

The effective arm length can simply be written as

$$l_2 = xl_1 = \frac{n_1 + n_2}{2 \sin(\alpha)} \quad (14)$$

Figure (2) shows the tracking error as function of the play radius.

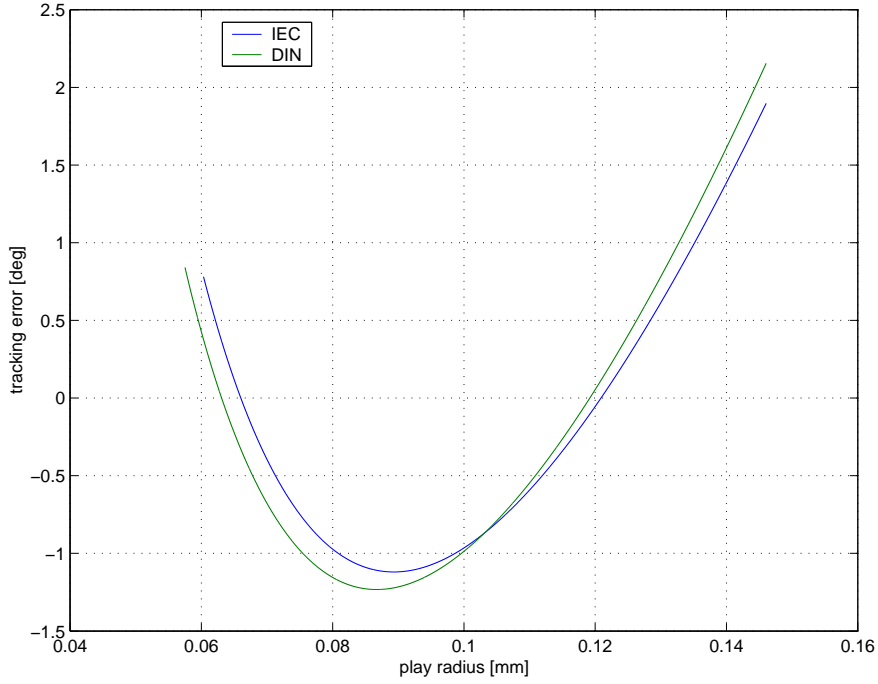


Figure 2: Tracking error according to known theory with an offset cantilever angle of 24 degrees.

3.2 Headshell offset taken into account

Here, the stylus can be moved along two parallel sleeves in the headshell. This has little effect on the effective arm length and on the offset angle. The example from the previous section will be used as a starting point. There are two properties to be resolved, a and b where Δ is initially set to zero. This property is introduced to characterize the effect of moving the stylus ± 5 mm or so.

From Eq. (7) it is clear that

	a [mm]	b [mm]	l_1 [mm]
IEC	93.45	209.88	211.67
DIN	90.14	204.69	206.60

If one plays with the sleeve distance, one will get an error as depicted in Fig. 3.

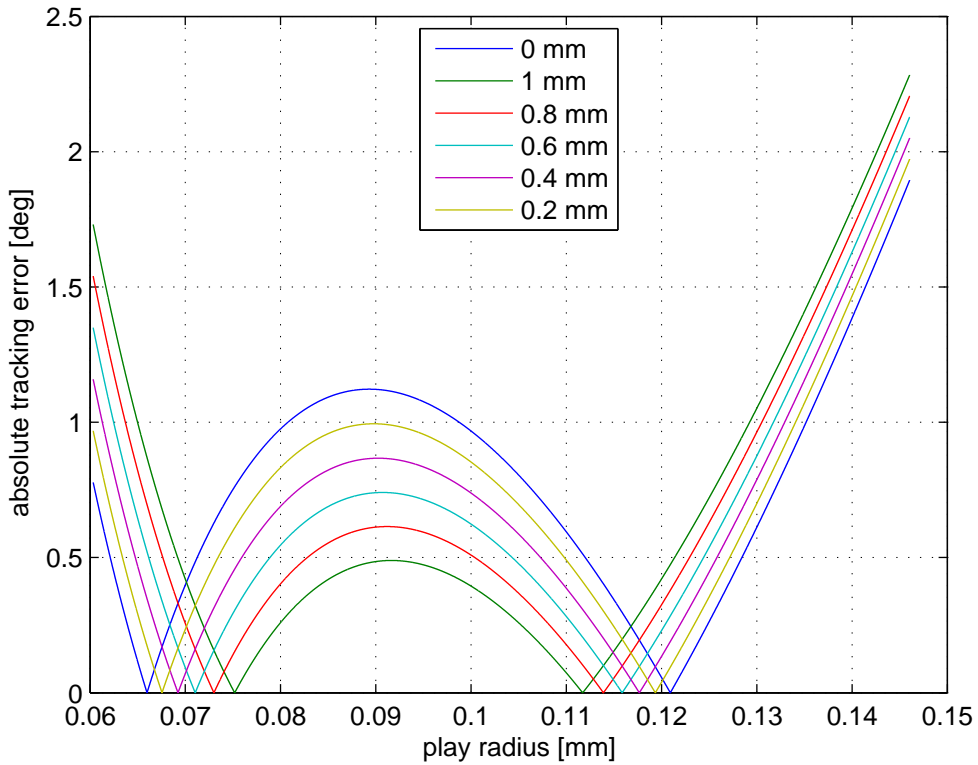


Figure 3: Various headshell sleeve offsets (Δ), IEC only.

In this particular case, an additional 0.2 mm keeps the error below 1 degree.

4 An example with an offset of 22 degrees

First, let us consider the method of peak distortion equivalence, where two zero's can be calculated independent of tone arm properties.

	α [deg]	l_1 [mm]	x	l_2 [mm]
IEC	22	232.91	1.0710	249.44
DIN	22	227.30	1.0703	243.28

My own turntable (AKAI AP-001) has $l_1 = 205$ mm, $l_2 = 218$ mm and $\alpha = 22$ degrees. It seems that this turntable is tuned differently. In a negative way, because the tone arm length should be larger than what is physically possible. Shorter tone arms do have a larger tracking error by definition. As a starting point the situation is chosen that the stylus is placed in the headshell such that the effective arm length is the shortest, or 218 mm. The properties a and b can be calculated and they are:

a [mm]	b [mm]
81.7	202.1

Here, a straightforward method can be applied. That is, plotting the tracking error as function of the effective arm length (l_2). By the way, this method can always be applied instead of first determine the zero's in case of a fixed offset and a fixed pivot to spindle distance. Fig. (4) shows the tracking error as function of effective arm length.

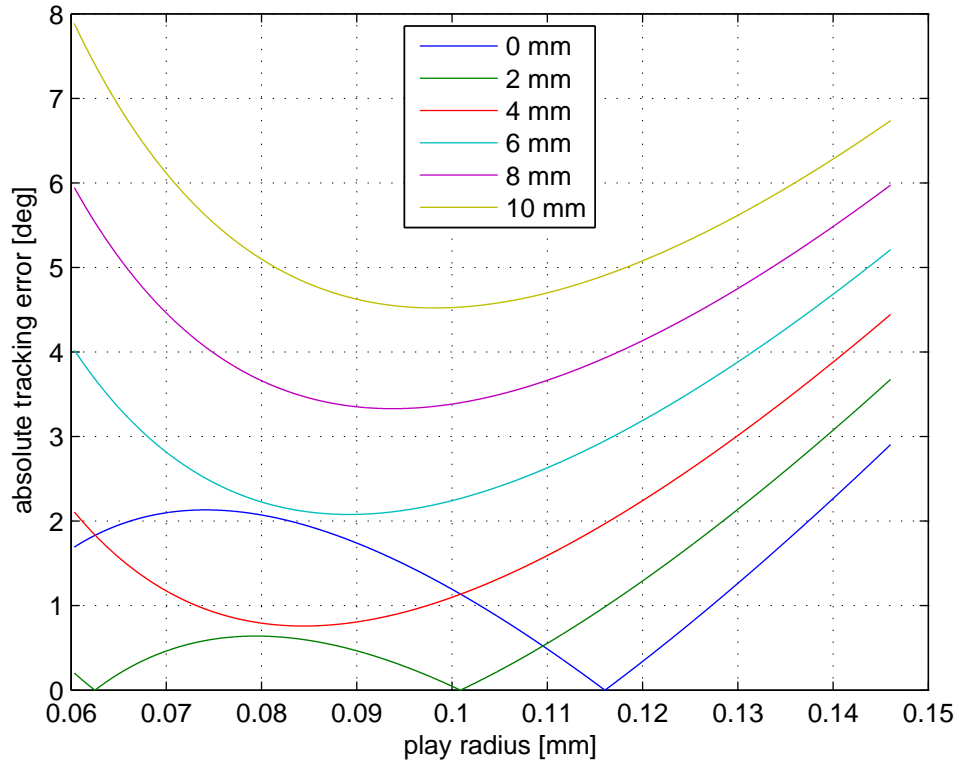


Figure 4: Absolute tracking error comparison with an offset cantilever angle of 22 degrees. The various curves are corresponding with the offset distance Δ .

It is clear that this tone arm produces (much) more harmonic distortion than the one used in the 24 degrees example. This is mainly due to the better geometry of the 24 degree tone arm.

Suppose that one chooses to use the AKAI tone arm with a headshell offset of 2 mm, how does the protractor look like to achieve this? According to Eq. (4) one obtains two zero's: 62.8 mm and 100.6 mm. This in contrast to the "standard" zero's.